

Entanglement sudden death in qubit-qutrit systems under depolarizing noise

Salman Khan*

Department of Physics, Quaid-i-Azam University,
Islamabad 45320, Pakistan

December 7, 2010

Abstract

I study the effects of decoherence on some qubit-qutrit systems under the influence of global, collective, local and multilocal depolarizing noise. I show that the entanglement sudden death (ESD) can be avoided under particular situations. The conjecture that ESD occurs in all bipartite states is questioned. I show that a critical point exists at which all the states are equally entangled. Furthermore, no ESD occurs when only the qubit is coupled to its local environment.

PACS: 03.65.Ud; 03.65.Yz; 03.67.Mn; 03.67.Pp

Keywords: Entanglement; Decoherence; qutrits.

1 Introduction

Quantum entanglement is one of the potential sources of quantum theory. It plays a vital role and works as a major resource for quantum communication and computation [1]. Unveiling various aspects of quantum entanglement has been a strenuous exercise for the last few decades. It is frequently used in constructing many protocols, such as teleportation of unknown states [2], quantum key distribution [3], quantum cryptography [4] and quantum computation [5, 6]. Completely isolating a quantum system from its environment is plainly an impossible task. It is, thus, necessary to investigate the behavior of initial state entanglement in the presence of environmental effects. The inevitable coupling between an environment and a quantum system leads to the phenomenon of decoherence and it gives rise to an irreversible transfer of information from the system to the environment [7, 8, 9]. Yu and Eberly [10, 11] showed that entanglement loss occurs in a finite time under the action of pure vacuum noise in a bipartite state of qubits. They found that, even though it takes infinite time to complete decoherence locally, the global entanglement may be lost in

*sksafi@phys.qau.edu.pk

finite time. This phenomenon of sudden loss of entanglement has been named as "entanglement sudden death" (ESD). The finite time loss of entanglement definitely limits the application of entangled states in quantum information processing. The phenomenon of ESD is not limited only to two qubit entangled states, it is investigated in systems of larger spaces such as qutrits and qudits [12, 13, 14, 15, 16, 17, 18, 19]. A geometric interpretation of the effect of ESD is given in Ref. [20]. The experimental evidences of the phenomenon of ESD have been reported for optical setups [21] and atomic ensembles [22].

Quantum states are grouped into separable and entangled states for qubit-qubit and qubit-qutrit system by using Peres-Horodecki criterion [24, 25], nevertheless, such characterization for higher dimensional bipartite states is difficult [26]. According to this criterion, the partial transpose of a separable density matrix must have non-negative eigenvalues. For a nonseparable state, the degree of entanglement is quantified by the negativity, which is given by the sum of the absolute values of the negative eigenvalues of the partial transpose of the density matrix.

In this paper I study the behavior of entanglement of a hybrid qubit-qutrit system in the presence of depolarizing noise. The individual qubit and qutrit states of the system, I consider, are incoherent but the composite system may still possess coherence and entanglement. For detailed study of the properties of such systems, I refer the readers to Ref. [27], where the authors showed the existence of ESD in such composite systems under the influence of dephasing noise. I consider various coupling of the system and environment in which the system is influenced by global, collective, local, or multilocal depolarizing noise. I show that although ESD occurs only in certain states under particular situations for various coupling of the system and environment, however, it can be controlled and completely avoided if certain measurement are taken. For example, under the action of multilocal and global noise, all the states avoid ESD if one or the other environment is controlled. Furthermore, I show that the degree of entanglement regrows for all states in the range of large values of decoherence parameters.

1.1 Qubit-Qutrit System in Depolarizing Noise

I consider a composite system of a qubit A and a qutrit B that are coupled to a noisy environment both collectively and individually. The collective coupling refers to the situation when both the qubit and qutrit are influenced by the same environment, whereas the local and multilocal coupling describes the situation when the qubit and qutrit are independently influenced by its own environment. The system is said to be coupled to a global environment when it is influenced by both collective and multilocal noises at the same time. Let the bases of Hilbert space of the qubit be denoted by $|0\rangle$ and $|1\rangle$ and that of the qutrit by $|0\rangle$, $|1\rangle$ and $|2\rangle$. Then the bases of the composite system are given in the order $|00\rangle$, $|01\rangle$, $|02\rangle$, $|10\rangle$, $|11\rangle$, $|12\rangle$.

The dynamics of the composite system in the presence of depolarizing noise can best be described in the Kraus operators formalism. The Kraus operators

for a single qubit depolarizing noise are given as

$$\begin{aligned} E_o^A &= \sqrt{1-p}\sigma_0, & E_1^A &= \sqrt{p/3}\sigma_1, \\ E_2^A &= \sqrt{p/3}\sigma_2, & E_3^A &= \sqrt{p/3}\sigma_3, \end{aligned} \quad (1)$$

where σ_i are the Pauli matrices. The Kraus operators for a single qutrit depolarizing noise are given as [28]

$$\begin{aligned} E_0^B &= \sqrt{1-p}I_3, & E_1^B &= \sqrt{\frac{p}{8}}Y, & E_2^B &= \sqrt{\frac{p}{8}}Z, \\ E_3^B &= \sqrt{\frac{p}{8}}Y^2, & E_4^B &= \sqrt{\frac{p}{8}}YZ, & E_5^B &= \sqrt{\frac{p}{8}}Y^2Z, \\ E_6^B &= \sqrt{\frac{p}{8}}YZ^2, & E_7^B &= \sqrt{\frac{p}{8}}Y^2Z^2, & E_8^B &= \sqrt{\frac{p}{8}}Z^2, \end{aligned} \quad (2)$$

with

$$Y = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad (3)$$

where $\omega = e^{2i\pi/3}$ and I_3 is the identity matrix of order 3. In Eqs. (1) and (2) $p = [0, 1]$ is the decoherence parameter. The lower and upper limits of p stand, respectively, for undecohered and fully decohered cases of the noisy environment. The Kraus operators for both qubit and qutrit satisfy the completeness relation $\sum_i E_i^\dagger E_i = I$. The evolution of the initial density matrix of the system when it is influenced by the global depolarizing noise is given in the Kraus operators formalism as follow

$$\rho' = \sum_{i,j,k} (E_i^{AB} E_j^B E_k^A) \rho (E_k^{A\dagger} E_j^{B\dagger} E_i^{AB\dagger}), \quad (4)$$

where $E_k^A = E_m^A \otimes I_3$, $E_j^B = \sigma_0 \otimes E_n^B$ are the Kraus operators of the multilocal coupling of qubit and qutrit individually and E_i^{AB} are the Kraus operators of the collective coupling that are formed from the tensor product of the Kraus operators of a single qubit and a single qutrit depolarizing noise in the form $E_m^A \otimes E_n^B$. The subscripts $m = 0, 1, 2, 3$, and $n = 0, 1, 2, \dots, 8$ stand for a single qubit and a single qutrit Kraus operators of a depolarizing noise. The initial density matrix ρ of the composite system is given by the following one parameter family of matrices

$$\rho(x) = \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & x \\ 0 & \frac{1}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{8} & 0 \\ x & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}, \quad (5)$$

where $0 \leq x \leq \frac{1}{4}$. For this range of values of x , all the eigenvalues are positive and the matrix is a well-defined density matrix. The partial transpose criterion for this matrix shows that in the specified range of x , it is entangled (see Ref. [27]). Using initial density matrix of Eq. (5) in Eq. (4) and taking the partial transpose over the qubit, it is easy to find the eigenvalues. Let the decoherence parameters for local noise of the qubit and qutrit and collective noise of the composite system be p_1 , p_2 and p respectively. Then, the eigenvalues of the partial transpose of the final density matrix when only the qubit is coupled to the noisy environment are given by

$$\begin{aligned}\lambda_{1,2} &= \frac{1}{4} - \frac{3}{32}p_1, \\ \lambda_{3,4} &= \frac{1}{8} + \frac{3}{64}p_1, \\ \lambda_5 &= \frac{1}{8} + \frac{3}{64}(1 - 24x)p_1 + x \\ \lambda_6 &= \frac{1}{8} + \frac{3}{64}(1 + 24x)p_1 - x.\end{aligned}\tag{6}$$

The only possible negative eigenvalue is the last one. The degree of entangle-

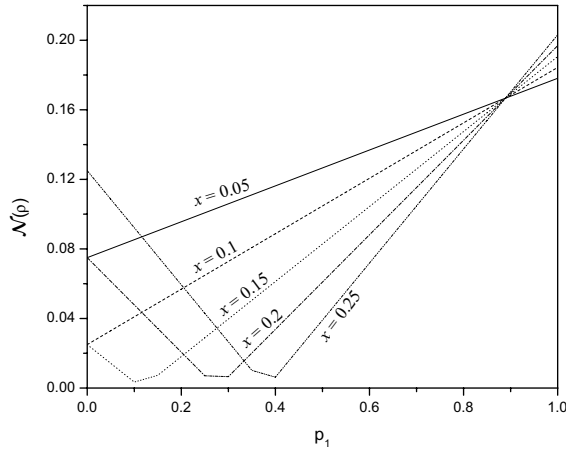


Figure 1: The negativity for the case when only the qubit is coupled to the environment is plotted for different density matrices against the decoherence parameter p_1 .

ment according to the definition of negativity becomes

$$\mathcal{N}(\rho) = \max \left\{ 0, \left| \frac{1}{8} + \frac{3}{64} (1 + 24x) p_1 - x \right| \right\}, \quad (7)$$

To observe the dynamics of entanglement, I plot the negativity for various values of x , which represents different density matrices, against the decoherence parameter p_1 in Fig. 1. The figure shows that the behavior of negativity splits the density matrices under consideration into two groups. In one case, the negativity increases linearly with increasing values of p_1 . In the other case, the negativity first decreases to a minimum for a particular value of p_1 and then increases linearly as p_1 increases. However no ESD occurs for the whole range of decoherence parameter for any density matrix. There is a critical point at which, irrespective of the values of x , all the density matrices reach to the same degree of entanglement and it happens at $p_1 = 0.888$. Beyond this value of p_1 , the degree of entanglement is higher for density matrices that correspond to large values of x .

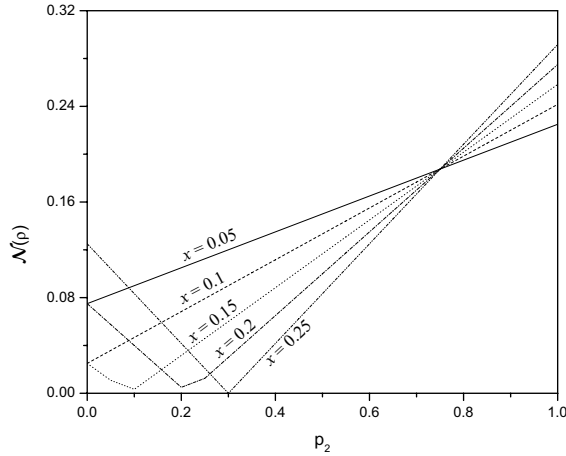


Figure 2: The negativity for the case when only the qutrit is coupled to the environment is plotted against the decoherence parameter p_2 for different values of the parameter x .

The eigenvalues of the partial transpose matrix when only the qutrit is cou-

pled to local depolarizing noise are given by

$$\begin{aligned}
\lambda_{1,2} &= \frac{1}{4} - \frac{3}{32}p_1 - \frac{1}{12}p_2 + \frac{3}{32}p_1p_2 \\
\lambda_{3,4} &= \frac{1}{8} + \frac{3}{64}p_1, \\
\lambda_5 &= \frac{1}{8} + \frac{1}{12}(1 - 16x)p_2 + x, \\
\lambda_6 &= \frac{1}{8} + \frac{1}{12}(1 + 16x)p_2 - x.
\end{aligned} \tag{8}$$

The only eigenvalue that possibly become negative is the last one. The negativity in this case becomes

$$\mathcal{N}(\rho) = \max \left\{ 0, \left| \frac{1}{8} + \frac{1}{12}(1 + 16x)p_2 - x \right| \right\}. \tag{9}$$

The behavior of entanglement in this case is nearly similar to the case of only qubit undergoing decoherence. Besides the rate of change in variation of negativity, ESD does occur in the range of large values of x and the critical point at which all the density matrices are equally entangled shifts to $p_2 = 0.75$. Moreover, the degree of entanglement for large values of x for a fully decohered case is higher as compared to the case of only qubit decoherence. The negativity in this case is plotted against the decoherence parameter p_2 in Fig. 2.

The eigenvalues of the partial transpose of the final density matrix for the multilocal case becomes

$$\begin{aligned}
\lambda_{1,2} &= \frac{1}{12}(3 - p_2), \\
\lambda_{3,4} &= \frac{1}{8}, \\
\lambda_5 &= \frac{1}{8} + \frac{3}{64}p_1 + \frac{1}{12}p_2 - \frac{3}{32}(1 - 16x)p_1p_2 \\
&\quad - \left(\frac{9}{8}p_1 + \frac{4}{3}p_2 - 1 \right) x, \\
\lambda_6 &= \frac{1}{8} + \frac{3}{64}p_1 + \frac{1}{12}p_2 - \frac{3}{32}(1 + 16x)p_1p_2 \\
&\quad + \left(\frac{9}{8}p_1 + \frac{4}{3}p_2 - 1 \right) x.
\end{aligned} \tag{10}$$

Again, the only one possibly negative eigenvalue is the last one.

$$\mathcal{N}(\rho) = \max \left\{ 0, \left| \frac{1}{8} + \frac{3}{64}p_1 + \frac{1}{12}p_2 - \frac{3}{32}(1 + 16x)p_1p_2 + \left(\frac{9}{8}p_1 + \frac{4}{3}p_2 - 1 \right) x \right| \right\}. \tag{11}$$

The negativity for multilocal noise is plotted in Fig. 3 against the decoherence parameter p_1 for $p_2 = 0.3$. It can be seen from the figure that no density matrix

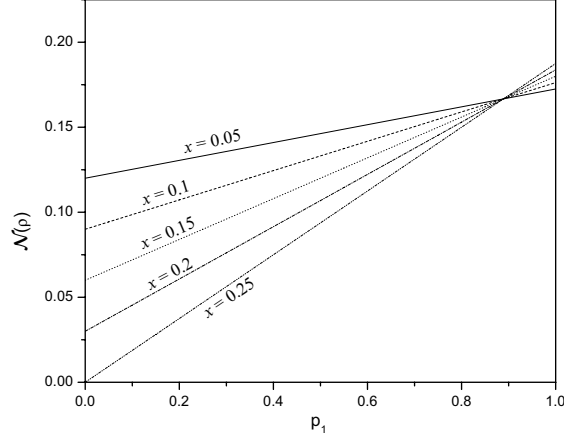


Figure 3: The negativity for the case of multilocal noise is plotted against the decoherence parameter p_1 for various values of the parameter x for $p_2 = 0.3$.

in the chosen range of x , undergoes ESD. It is true for $0.3 \leq p_2 < 1$. For $p_2 < 0.3$, the ESD occurs not for all but only for certain density matrices in the upper limit of x . The degree of entanglement increases for density matrices of large x as p_1 increases when $0.32 \leq p_2 \leq 0.6538$. Whereas for $0.6538 \leq p_2 < 1$, the degree of entanglement decreases with increasing values of p_1 . This is shown in Fig. 4. A nearly similar behavior of the negativity is observed when p_1 is kept constant.

For collective noise the eigenvalues are obtained by replacing $p_1 = p_2$ in Eq. (10). In this case the negativity becomes

$$\mathcal{N}(\rho) = \max \left\{ 0, \left| \frac{1}{8} + \frac{1}{32} \left(\frac{25}{6} - 3p \right) p \left(\frac{59}{24} - \frac{3}{2}p \right) px - x \right| \right\}. \quad (12)$$

The effect of decoherence on the negativity is shown in Fig. 5. The behavior of negativity in this case is nearly similar as discussed previously. However, the ESD occurs only in the intermediate range of values of x . Unlike the previous cases, there are two critical points that happens at $p = 0.75, 0.888$. Beyond the first critical point the negativity for lower values of x decreases and for higher values of x first increases and then drops reaching the second critical point as p increases.

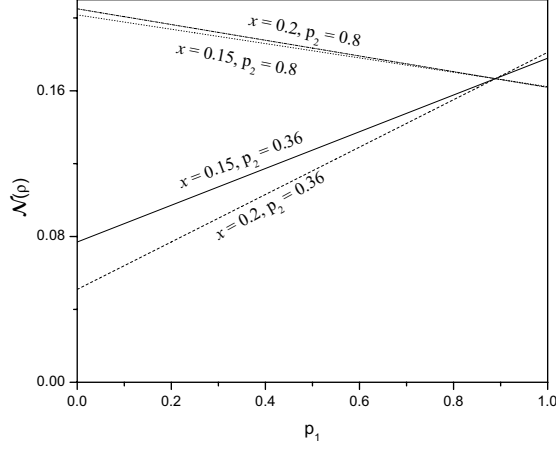


Figure 4: The negativity for the case of multilocal noise is plotted against the decoherence parameter p_1 for different set of values of decoherence parameter p_2 and the parameter x .

Finally, I consider the influence of global depolarizing noise. The general form of the eigenvalues of the partial transpose of the final density matrix for this case have quite lengthy expressions. Instead of writing the general expressions for all of them, I consider only the one that can possibly become negative, which is the last one as before. For a special case in which the multilocal decoherence parameters $p_1 = p_2 = 1/2$, it becomes

$$\lambda_6 = \frac{1}{4608} [96(8 - 7x) - 63p^2(1 + 16x) + 28p(2 + 59x)] \quad (13)$$

The corresponding negativity becomes

$$\mathcal{N}(\rho) = \max \left\{ 0, \left| \frac{[2184 - 4800x - 450p^2(1 + 16x) + 5p(107 + 2360x)]}{13824} \right| \right\}. \quad (14)$$

The behavior of negativity as a function of p for various values of x is shown in Fig. 6. It is clear from the figure that ESD can be completely avoided under the action of global environment for the chosen values of multilocal decoherence parameters. However, this is not true for all values of multilocal decoherence parameters. For example, for $p_1 = p_2 = 1/10$, some density matrices in the

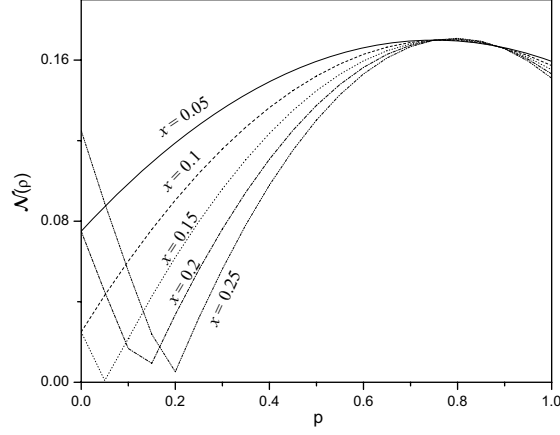


Figure 5: The negativity for the case of collective noise is plotted against the decoherence parameter p for various value of the parameter x .

range of large values of x exhibit ESD. Furthermore, for $p_1 = p_2 = 3/4$, the negativity becomes independent of x and all the state respond equally to p .

2 Summary

I study the dynamics of entanglement for certain hybrid qubit-qutrit states under global, collective, local and multilocal depolarizing noise. I show that unlike the case of dephasing noise [27], the influence of depolarizing noise is different for different coupling of the system and the environment. Using partial transpose criterion for quantifying entanglement, it is shown that not all but only certain initially entangled density matrices exhibit ESD in depolarizing noise in the range of lower values of decoherence parameter under particular coupling. No ESD in any density matrix occurs when only the qubit is coupled to its local environment. The only density matrices that corresponds to higher values of x exhibit ESD when only the qutrit is coupled to its local environment. The conjecture made in Ref. [27] that ESD is a generic phenomenon to occur in all bipartite quantum system is not correct. For every density matrix that undergoes ESD at a particular value of decoherence parameter, the re-birth of entanglement occurs at values higher than that particular value of decoherence

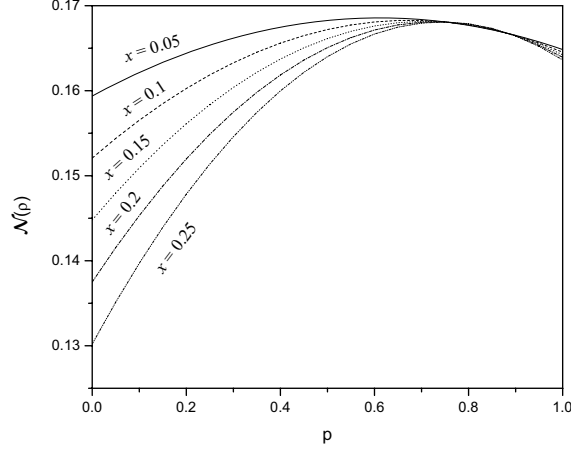


Figure 6: The negativity for the case of global noise when $p_1 = p_2 = 0.5$ is plotted against the decoherence parameter p for various values of the parameter x .

parameter. The degree of entanglement increases for large values of decoherence parameter for all states under every possible coupling of the system and the environment considered here. A critical point, at which all the states has equal degree of entanglement, is observed. It is shown that in the case of only qutrit-environment's coupling the decrease in negativity occurs faster in certain density matrices as compared to only the qubit-environment's coupling. The entanglement sudden death happens only in the intermediate values of x under the action of collective noise. Furthermore, it is shown that the ESD can be completely avoided under certain situations when the system is under the action of multilocal and global noise.

References

- [1] Bouwmeester D, Ekert A and Zeilinger A 2000 *The Physics of Quantum Information* (Springer-Verlag Berlin)
- [2] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and W K Wootters 1993 Phys. Rev. Lett. **70** 1895

- [3] Ekert A 1991 Phys. Rev. Lett. **67** 661
- [4] Bennett C H, Brassard G, Mermin N D 1992 Phys. Rev. Lett. **68** 557
- [5] Grover L K 1997 Phys. Rev. Lett. **79** 325
- [6] DiVincenzo D P 1995 Science **270** 255
- [7] Zurek W H *et al* 1991 Phys. Today **44** 36
- [8] Breuer H P and Petruccione F 2002 *The Theory of Open Quantum Systems* (Oxford University Press Oxford); Carmichael H 1993 *An Open Systems Approach to Quantum Optics* (Springer, Berlin)
- [9] Zurek W H 2003 Rev. Mod. Phys. **75** 715
- [10] Yu T and Eberly J H 2002 Phys. Rev. B **66** 193306; 2003 **68** 165322
- [11] Yu T and Eberly J H 2004 Phys. Rev. Lett. **93** 140404; 2006 Opt. Commun. **264** 393; 2006 Phys. Rev. Lett. **97** 140403; 2007 Quantum Inf. Comput. **7** 459; 2009 **323**, 598.
- [12] Yonac M *et al* 2006 J. Phys. B **39** S621
- [13] Jakobczyk L and Jamroz A 2004 Phys. Lett. A **333** 35; Jamroz A 2006 J. Phys. A **39** 7727; Cui H T *et al* 2007 Phys. Lett. A **365** 44; Zhang G-F *et al* 2007 Opt. Commun. **275**, 274
- [14] Ikram M, Li F L, and Zubairy M S 2007 Phys. Rev. A **75** 062336
- [15] Al-Qasimi A and James D F V 2008 Phys. Rev. A **77** 012117
- [16] Ann K and Jaeger G 2007 Phys. Rev. A **76** 044101
- [17] Ann K and Jaeger G 2008 Phys. Lett. A **372** 579
- [18] Huang J H and Zhu S Y 2007 Phys. Rev. A **76** 062322
- [19] Jaeger G and Ann K 2007 J. Mod. Opt. **54** (16) 2327
- [20] Terra M O, Cunha 2007 New J. Phys. **9** 237
- [21] Almeida M P *et al* 2007 Science **316** 579; Salles A, *et al* 2008 Phys. Rev. A **78** 022322
- [22] Laurat J *et al* 2007 Phys. Rev. Lett. **99** 180504
- [23] Bennett C H *et al* 1996 Phys. Rev. Lett. **76** 722
- [24] Peres A 1996 Phys. Rev. Lett. **77** 1413
- [25] Horodecki M, Horodecki P, and Horodecki R 1996 Phys. Lett. A **223** 1

- [26] Horodecki R, Horodecki P, Horodecki M, and Horodecki K, 2009 Rev. Mod. Phys. **81** 865
- [27] Ann K, Jeager G 2008 Phys. Lett. A **372** 579.
- [28] Salimi S, Soltanzadeh M M, 2009 International Journal of Quantum Information **7** 615